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Stochastic Damage Growth Model for Fuselage Splices with Multisite Damage

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Research has been conducted on methods for risk assessment of fuselage splice joints containing multisite damage (MSD), and extensive test data from MSD specimens have been obtained. The objective is to propose a test data-based methodology for probabilistic analysis of lap splices with MSD. In the probabilistic analysis, the failure characteristics of nine noncorroded MSD specimens under fatigue loading were examined, and a failure criterion was proposed involving an aggregated lead crack in a segment where the first linkup occurred. A curve-fitting technique was used to characterize the growth behavior of the aggregated lead crack in each specimen. The total fatigue life of each specimen was divided into two stages: damage starting and damage growth. The starting life was assumed to follow a lognormal distribution derived from the test data. In the growth stage, a stochastic damage growth model was developed based on the censored test data. This model, along with the derived distribution of the visible damage starting life, was used to predict the probability of failure for the MSD specimens tested. Comparisons between the data from analysis and test were made and are presented to demonstrate the effectiveness of the developed model.

Nomenclature

a	=	length of aggregate crack
a_{cr}	=	critical crack length
$f_{N0}(\cdots)$	=	probabilistic density function of visible damage starting life
K_{max}	=	maximum stress intensity factor
N	=	number of load cycles
N_g	=	damage growth life
N_t	=	total fatigue life
N_0	=	visible damage starting life
$\tilde{N}_0$	=	median value of observed visible damage starting life
$P(\cdots)$	=	cumulative distribution function
Q, b	=	deterministic constants in damage growth function
q	=	logarithm of Q
R	=	stress ratio of loading spectrum
U	=	logarithm of crack length
u_i, v_i	=	random numbers between zero and unity
X	=	lognormal random variable associated with damage growth rate
Y	=	logarithm of damage growth rate
Z	=	normal random variable associated with damage growth rate
Z_0	=	normal random variable associated with visible damage starting life
ΔK	=	stress intensity factor range
μ_{N0}	=	mean value of visible damage starting life
μ_Y	=	mean value of Y
μ_Z	=	mean value of Z
μ_{Z0}	=	mean value of Z_0
σ_{N0}	=	standard deviation of visible damage starting life
σ_Y	=	standard deviation of Y
σ_Z	=	standard deviation of Z
σ_{Z0}	=	standard deviation of Z_0
$\Phi(\cdots)$	=	standard normal distribution function

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Introduction

FATIGUE crack growth analysis is one of the major tasks in dealing with the structural integrity and damage tolerance of fatigue critical structural components in aging aircraft. Risk analysis of fatigue critical structures has recently become an important issue to be addressed, and various methodologies have been developed (e.g., see Refs. 1–3). A longitudinal skin splice is a fatigue critical structure in which multiple site fatigue damage (MSD) may develop under fatigue loading due to repeated pressurization. Because of the considerable statistical variability in fatigue crack propagation, the investigation of the failure characteristics of lap splices involving MSD using probabilistic analysis methods is receiving wide attention.[4–8]

This paper focuses on the development of a stochastic damage growth model to predict the failure probability of fuselage splices involving MSD. The work is based on the test data of nine noncorroded MSD specimens, which have been obtained in an MSD test program[9] at the Institute for Aerospace Research, National Research Council. Berens et al.[10] have used these data to verify their probabilistic analysis using the code PROF, which is based on a deterministic crack growth model and equivalent initial flaw size distributions. This type of probabilistic analysis relies largely on the efficiency of the deterministic crack growth analysis using sophisticated finite element methods and the accuracy of the derived equivalent flaw size distributions. Because of the complexity of MSD problems, it is very difficult to consider all possible damage scenarios and their interactions if there are no efficient analysis models for prediction of crack propagation. From the standpoint of practical applications, the probabilistic analysis methodologies should be as simple as possible while maintaining reasonable accuracy for prediction of the failure probability of fatigue critical components.

The objective of the present work is to develop a test data-based methodology that can be used to predict the probabilistic characteristics of lap splices with MSD, including the percentiles and the distribution of crack size at any given time and, more important, the probability of failure. Following the procedures proposed by Yang et al.[11] for single fastener hole specimens, a stochastic damage growth model is established for lap splices with MSD. In this model, sophisticated fracture mechanics analysis for each specimen at every growth time is not required, so that the approach is less complex than risk analysis using deterministic crack growth prediction and distributed initial flaw sizes. Because all stochastic characteristics are inherently included in the test data, the model based on test data might also be expected to provide a more reasonable assessment of failure probability. In addition, provided sufficient test data are available, this modeling approach can be used to deal, in a relatively simple manner, with the problems of fatigue and corrosion interactions in splices. Only the analysis results using noncorroded MSD specimens are presented.

Failure Characteristics of MSD Specimens

The structural component under consideration is a fuselage single lap splice joint, as shown in Fig. 1. Test specimens have been specially designed to simulate aircraft conditions, in particular the development of MSD and the MSD failure mode. The key feature of the concept is the use of bonded side straps to simulate the load transfer from cracked areas to the surrounding structure that occurs on aircraft. This allows MSD cracks to develop and linkup in a typical manner, with typical crack growth rates, and without premature failure of the specimen. A shaped doubler at each end of the specimen is used to fine tune the hoop stress distribution. Nine noncorroded MSD specimens were tested under a constant amplitude far-field stress of 12.9 ksi (88.9 MPa) with a load ratio of $R = 0.2$ and a frequency of 8 Hz (Ref. 9). The historical data of crack size vs load cycles for all cracks, which initiated in the top row of rivet holes, were obtained for probabilistic analysis in the present work.

The specimens were constructed of two 2024-T3 clad aluminum sheets with a thickness of 0.04 in. (1 mm) and a width of 10 in. (25.4 cm). Each of these sheets was connected to a doubler with a thickness of 0.16 in. (4 mm). There were three rows of 0.16 in. (4 mm) 2117-T4 rivets (MS20426AD5-5) with eight rivets in each row. The rivet pattern had 1-in. (25.4-mm) pitch and row spacing with an edge margin of 0.36 in. (9.1 mm). The edges of the spliced sheets were reinforced by straps that were bonded together.

The raw test data for the nine specimens were used to derive the crack growth histories at all rivet holes. Examination of the historical data showed that visible cracks initiated in different scenarios from one or two sides of a single hole or multiple rivet holes located in the central part of the specimens. The cracks spread with the load cycles from the central holes outward forming a pattern of MSD. There was one ligament between two holes in each specimen, where the two cracks initially linked up or the crack tips coalesced. After the failure of the first ligament, crack growth became unstable, resulting in the final failure of the whole specimen. The first linkup and the final break are the two primary failure modes of the MSD specimens.

In this paper, the specimens are considered to have failed when the first linkup occurs. Table 1 shows the fatigue life of the MSD specimens corresponding to the first linkup and the final break. It can be seen that the difference between the fatigue life of the two failure modes is less than 4%. Therefore, it is treasonable to consider the first linkup as a failure criterion for those specimens under investigation. Further research will be conducted at a later stage to examine the effects of various failure criteria.

The focus of this paper is on a probabilistic analysis methodology for risk assessment of the MSD specimens. For the sake of simplicity, only one ligament in each specimen where the first linkup occurred is considered in the subsequent discussions. The two cracks in this ligament are aggregated and are treated as a single crack called the aggregated lead crack. The failure criterion states that the specimen fails when the length of the aggregated lead crack a reaches the critical value a_{cr}, which is the length of the ligament. The critical value for the nine MSD specimens under consideration is $a_{cr} = 0.844$ in. (21.4 mm).

The total service period of a damaged structure is usually divided into two stages: damage initiation and damage growth. Damage initiation is a complex process involving microstructural behavior. This process is not detectable in a rivet lap splice using visual means because it occurs under the rivet head. When the damage becomes visible, it has already experienced a short growth period from initiation to a visible size. From an engineering point of view, it is appropriate to consider the total service period of a lap splice as two portions: starting stage and growth stage of visible cracks. That is, the total fatigue life is distinguished as starting life N_0, growth life

Table 1 Fatigue life of MSD specimen at first linkup and final break

	Fatigue life (cycles)		
Specimen	First linkup	Final break	Difference, %
Cgc-f0	405,902	412,575	1.6
Cgc-f1	362,500	368,335	1.6
Cgc-f3	228,500	231,050	1.1
Cgc-f4	255,600	260,400	1.9
Cgc-f5	173,452	176,957	2.0
Cgc-f6	436,940	441,333	1.0
Cgc-f7	250,800	259,180	3.3
Cgc-f8	292,700	298,550	2.0
Cgc-f9	199,850	203,075	1.6

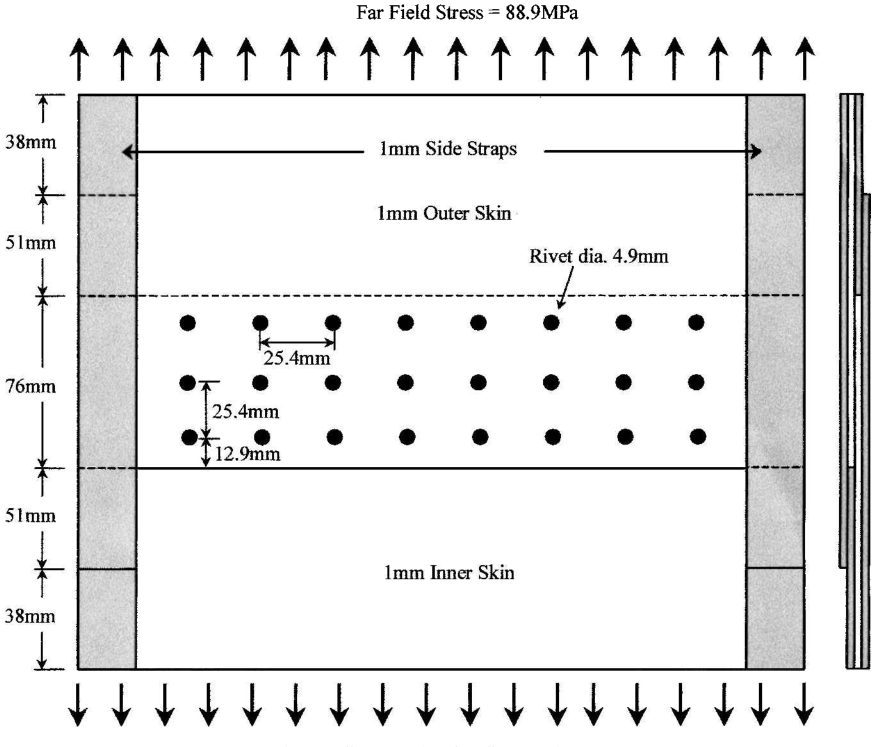

Fig. 1 Schematic of MSD specimen.

Table 2 Initiation, growth, and total life of MSD specimen

Specimen	Fatigue life (cycles) Starting	Growth	Total
Cgc-f0	336,563	69,339	405,902
Cgc-f1	291,955	70,545	362,500
Cgc-f3	188,500	40,000	228,500
Cgc-f4	195,000	60,600	255,600
Cgc-f5	133,000	40,452	173,452
Cgc-f6	385,000	51,940	436,940
Cgc-f7	201,003	49,797	250,800
Cgc-f8	225,000	67,700	292,700
Cgc-f9	152,000	47,850	199,850

Table 3 Initial flaw size, initiation life, and growth life of MSD specimen

Specimen	Original measurements: Size at first observation, mm	Original measurements: Cycles at first observation	Fitted data after censoring: Size at 5000 cycles, mm	Fitted data after censoring: Cycles at 0.068-in. crack
Cgc-f0	2.438	336,563	2.007	329,680
Cgc-f1	1.194	291,955	2.083	301,500
Cgc-f3	1.397	188,500	2.311	192,730
Cgc-f4	3.200	195,000	1.981	177,600
Cgc-f5	1.854	133,000	2.235	127,900
Cgc-f6	1.829	385,000	2.159	383,750
Cgc-f7	2.515	201,003	2.083	185,150
Cgc-f8	1.422	225,000	2.083	231,180
Cgc-f9	1.727	152,000	2.210	153,660

N_g, and total life N_t, that follow $N_t = N_0 + N_g$. The visible damage starting life is the number of load cycles at which the first crack from under the rivet head was observed, and the total life is that of a specimen at which the first linkup occurred. The growth life is the difference between the total life and the starting life. The three types of fatigue lives of the nine specimens are shown in Table 2. The test data shown in this table will be used in the following discussions for verification of the analysis model.

The initial visible flaw size of the aggregated lead crack and the damage starting life of the nine specimens from the original measurements are shown in Table 3. It is observed from Tables 1–3 that the nine specimens started visible cracks at a life between 133,000 and 385,000 cycles and failed at a life between 173,452 and 436,940 cycles. The statistical dispersion of the crack starting and growth damage accumulation is large, a typical phemenon of MSD specimens.

Stochastic Damage Growth Model

Because the MSD test data show only the growth behavior of visible cracks, the theoretical development in this section deals with the stochastic characteristics in the damage growth period. Damage initiation behavior is not considered in detail. However, the visible damage starting life of each specimen will be taken into account in predicting the total life of the specimens. In investigating damage growth behavior, an initial crack is assumed to exist in each specimen. It is reasonable to take the median of the nine initial visible crack lengths, which is 0.068 in. (1.73 mm), to be the length of the initial crack for all specimens. The number of load cycles of each specimen at which the aggregated crack length reaches this specified initial crack length is taken as the damage starting life, and a distribution for the starting life of all nine specimens is established.

Fatigue Crack Growth Data and Visible Crack Starting Life Distribution

To demonstrate the statistical variability of the crack growth damage accumulation, the crack propagation histories of the nine MSD specimens are plotted in Fig. 2. An exponential growth function available in Microsoft Excel is used to fit the nine sets of data, and the fitted curves are also shown in Fig. 2.

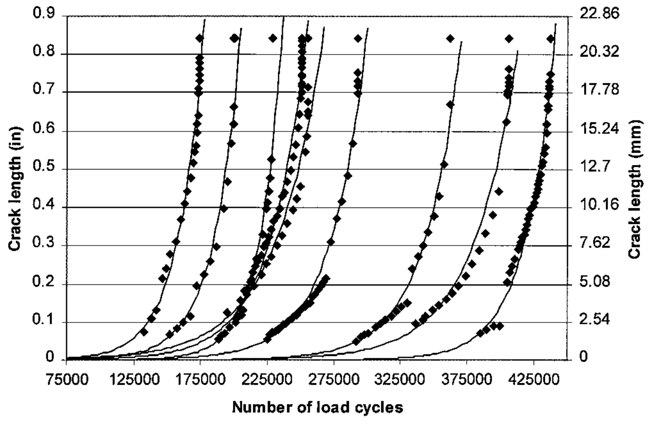

Fig. 2 Crack propagation histories with fitted curves.

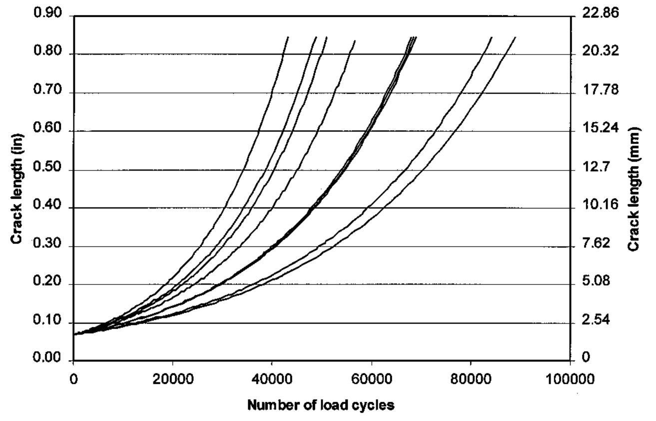

Fig. 3 Fitted curves after censoring.

To focus on the damage growth behavior, the data are censored to obtain homogeneous data sets of damage growth in which the growth curves of all specimens start with the same initial crack size. The fitted growth functions are used to determine the number of load cycles for each specimen at which the crack reaches 0.068 in. (1.73 mm). Then both the test data and the fitted data are normalized to zero cycle at this initial crack length. The fitted curves after censoring are shown in Fig. 3.

The visible starting life and the starting flaw size determined using the fitted growth functions are shown in Table 3. Using the commercial software BestFit, the damage starting life of the nine specimens, $N_0(\mu_{N0}, \sigma_{N0})$ is found to be represented the best by a lognormal distribution among the commonly used distributions in engineering. The mean value and the standard deviation of N_0 are calculated as $\mu_{N0} = 231{,}461$ cycles and $\sigma_{N_0} = 87{,}370$ cycles, respectively, which are close to the values determined by BestFit as $\mu_{N0} = 231{,}290$ cycles and $\sigma_{N0} = 82{,}991$ cycles, respectively. The natural logarithm of N_0 is a normal random variable $Z_0(\mu_{Z0}, \sigma_{Z0})$ as

$$Z_0 = \ln(N_0) \tag{1}$$

where μ_{Z0} are σ_{Z0} the mean value and the standard deviation of Z_0, respectively, which are calculated as $\mu_{Z0} = 12.29$ and $\sigma_{Z0} = 0.369$.

The probabilistic density function and the cumulative distribution function of the lognormal variable N_0 can be written as

$$f_{N0}(N) = \frac{1}{\sqrt{2\pi}\sigma_{Z0}N}\exp\left[-\frac{(\ln N - \mu_{Z0})^2}{2\sigma_{Z0}^2}\right] \tag{2}$$

$$P(N_0 \le N) = \Phi\left[\frac{\ln(N/\tilde{N}_0)}{\sigma_{Z0}}\right], \qquad \tilde{N}_0 = \frac{\mu_{N0}^2}{\sqrt{\mu_{N0}^2 + \sigma_{N0}^2}} \tag{3}$$

where $\tilde{N}_0$ median value of the number of load cycles corresponding to the initial crack length, which is calculated as $\tilde{N}_0 = 216{,}547$ cycles. The assumed lognormal distribution for damage starting life

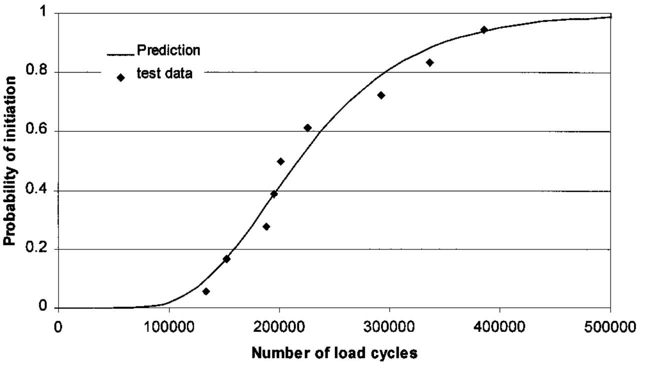

Fig. 4 Damage starting life distribution.

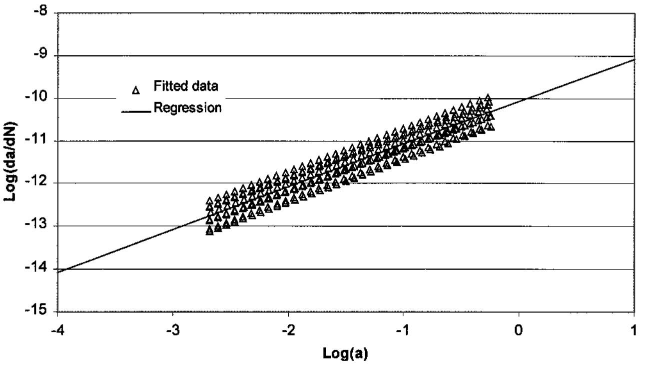

Fig. 5 Linear regression of log crack size and log crack growth rate.

is plotted in Fig. 4 along with the observed cumulative distributions of the load cycles at which the first visible cracks were observed during testing. The observed cumulative distribution function was obtained by ordering the cycles at the first observed visible cracks and dividing the ranks minus 0.5 of the ordered times by the sample size. The symmetric rank formula, that is, $P = (i - 0.5)/n$, is used in which i is the rank of the ith data point in the ordered set of n points. This formula is also used in the following sections to calculate the cumulative distributions of the observed growth and failure life. The comparison in Fig. 4 shows a good agreement between the assumed distribution curve and the observed data. It is also indicated that the damage starting life of the specimen has a significant statistical dispersion. A goodness-of-fit test is carried out for the fit shown in Fig. 4 with the Anderson–Darling (A–D) statistic calculated as $A_n^2 = 0.24$ following Eq. (9.1.6) of Lawless,[12] and from Table 9.2.5 of Ref. 12 it follows that the level of significance is well above 0.25.

Stochastic Crack Propagation Model

This section focuses on the growth behavior of the aggregated lead crack and presents the development of a stochastic damage growth model using the test data. For deterministic analysis, various fatigue crack growth functions have been proposed in the literature (e.g., see Miller and Gallagher[13] and Hoeppner and Krupp[14]). The growth functions can be represented in a general form as

$$\frac{\mathrm{d}a(N_g)}{\mathrm{d}N_g} = f(\Delta K, K_{\max}, R, S, a) \tag{4}$$

in which $f(\cdots)$ is a nonnegative function, N_g the number of load cycles, $a(N_g)$ the crack size at N_g, and S the maximum stress level in the loading spectrum. For crack propagation in fastener holes under spectrum loading, Yang[15] proposed a simple growth function expressed as

$$\frac{\mathrm{d}a(N_g)}{\mathrm{d}N_g} = Q[a(N_g)]^b \tag{5}$$

where Q and b are deterministic constants that depend on the characteristics of the spectrum loading and the materials of specimens. These constants can be determined using crack growth historical data.

To deal with the statistical variability of the crack growth rate, Eq. (5) is randomized by introducing an additional factor $X(N_g)$, following Yang et al.,[11] as

$$\frac{\mathrm{d}a(N_g)}{\mathrm{d}N_g} = X(N_g)Q[a(N_g)]^b \tag{6}$$

where $X(N_g)$ is a nonnegative stationary stochastic process with a median value equal to unity. Thus, the deterministic crack growth function given by Eq. (5) represents the median crack growth rate behavior, and the random process $X(N_g)$ accounts for the statistical variability of the crack growth rate. Furthermore, $X(N_g)$ can be taken as a stationary lognormal random process.[11] If the lognormal random process is completely correlated at any two load cycles, $X(N_g)$ becomes a lognormal random variable X and, thus, Eq. (6) becomes

$$\frac{\mathrm{d}a(N_g)}{\mathrm{d}N_g} = XQ[a(N_g)]^b \tag{7}$$

This simplification has been shown by Yang et al.[11] to be attractive for practical applications where the test data are not plentiful. Hence, the stochastic crack growth function given in Eq. (7) is employed in the present analysis.

To determine the constants and the lognormal random variable in Eq. (7), the natural logarithm of both sides of Eq. (7) is taken as

$$Y = bU + q + Z(N_g) \tag{8}$$

where $Z = \ln[X]$ is a normal random variable with a zero mean value and the other variables are

$$Y = \ln\left[\frac{\mathrm{d}a(N_g)}{\mathrm{d}N_g}\right], \qquad U = \ln[a(N_g)], \qquad q = \ln[Q] \tag{9}$$

Because Z is a normal random variable, the log crack growth rate Y is also a normal variable at all crack size $a(N_g)$. Therefore, the mean value and the standard deviation of Y can be calculated as

$$\mu_Y = bU + q, \qquad \sigma_Y = \sigma_Z = \sum_{i=1}^{m} \frac{[Y_i - (bU_i + q)]^2}{m} \tag{10}$$

where m is the total number of data points that are used in the linear regression to determine the constants b and q.

The fitted crack growth functions are used to generate the data of log crack size and log crack growth rate. From each growth function 50 data points are calculated. These data points are plotted in Fig. 5 along with a straight line obtained from linear regression fitting the data points. The crack growth rate parameters b and Q, as well as the standard deviation of Z, are then obtained:

$$b = 0.99898, \qquad Q = 4.1784 \times 10^{-5}, \qquad \sigma_Z = 0.23369 \tag{11}$$

The relative error of this fit is reflected by the squared standard deviation, which is calculated as 5.5%. Now, the cumulative distribution function of X can be written as

$$P(X \le x) = \Phi\{[\ln(x) - \mu_Z]/\sigma_Z\} = \Phi[\ln(x)/\sigma_Z] \tag{12}$$

The statistical variability of the damage growth behavior of MSD specimens is characterized by the distribution function in Eq. (12). For the statistical characteristics of the total life, the two distribution functions in Eqs. (3) and (12) will be used.

Prediction and Correlation with Test Data

The proposed stochastic damage growth model and the starting life distribution are used in this section to predict the probability of failure of MSD specimens. Correlation with the test data is conducted for both the damage growth life and the total life. In the former case, a closed-form solution is used, and in the latter case, a Monte Carlo simulation is employed.

Growth Life Distribution

For damage growth behavior, the statistical distribution can be derived analytically. Noting that the damage growth stage starts with a zero cycle and a specified initial crack size, we can derive the crack length a at load cycle N_g by integrating Eq. (7) as

$$a(N_g) = a_0 \Big/ \left[1 - (b-1) X Q N_g a_0^{b-1}\right]^{1/(b-1)} \qquad (13)$$

where $a_0 = 0.068$ in. (1.27 mm) is the initial crack size. With this analytical expression, all probabilistic parameters related to the growth life such as the percentile and the distribution function of the crack size at any service life can be obtained. Here the growth life distribution when the lead crack reaches the critical crack length a_{cr} is calculated. The critical crack length for the aggregated lead crack is taken as the length of a ligament between two rivet holes, that is, $a_{cr} = 0.844$ in. (21.4 mm).

Letting $N_g(a_{cr})$ denote the time to reach the critical crack size, we can obtain this value from Eq. (13) by setting $a(N_g) = a_{cr}$ and $N_g = N_g(a_{cr})$, that is,

$$N_g(a_{cr}) = [1/(b-1) X Q]\left[a_0^{b-1} - a_{cr}^{b-1}\right] \qquad (14)$$

The distribution of $N_g(a_{cr})$ can be derived from that of X through transformation of Eq. (14) as

$$P[N_g(a_{cr}) \leq n] = P[X \geq \eta] = 1 - \Phi[\ln(\eta)/\sigma_Z] \qquad (15)$$

where

$$\eta = [1/(b-1) Q n]\left[a_0^{b-1} - a_{cr}^{b-1}\right] \qquad (16)$$

Equation (15) is now used to calculate the life distribution in the growth stage, and the prediction is shown in Fig. 6. Also shown in Fig. 6 are the growth life of the nine specimens obtained by taking the difference between the observed first linkup life and the corresponding observed starting life of each specimen. The goodness-of-fit test for the fit shown in Fig. 6 results in an A–D statistic of $A_n^2 = 0.47$, which indicates that the level of significance is well above 0.25. The good agreement between the predicted distribution and the test data demonstrates the efficiency of the stochastic damage growth model developed. The results shown in Fig. 6 also indicate that the damage growth life of the MSD specimens has a relatively small statistical dispersion as compared to that of the damage starting life shown in Fig. 4.

Total Life Distribution

To predict the probability of failure at a service life or load cycle, the total life to reach the given critical crack length a_{cr} needs to be considered, which can be written as

$$N_t(a_{cr}) = N_0 + N_g(a_{cr}) = N_0 + [1/(b-1) X Q]\left[a_0^{b-1} - a_{cr}^{b-1}\right] \qquad (17)$$

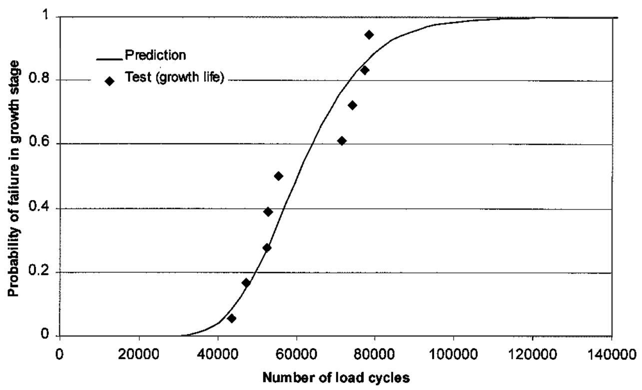

Fig. 6 Growth life distribution at failure ($a = a_{cr}$).

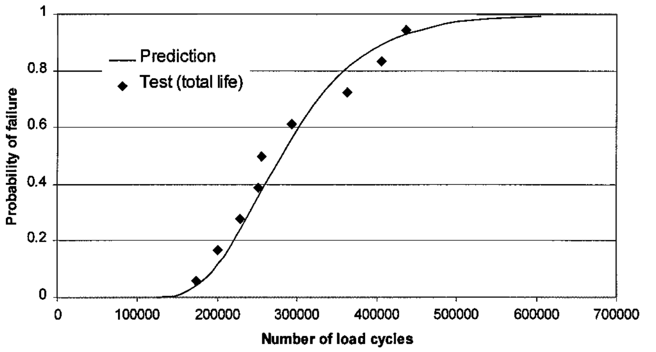

Fig. 7 Total life distribution of MSD specimens at failure (POF).

The probability of failure (POF) is the distribution of $N_t(a_{cr})$, which can be written as

$$\mathrm{POF}_{N_t(a_{cr})}(N) = P[N_t(a_{cr}) \leq N]$$

$$= P\left(N_0 + \{1/[(b-1) X Q]\}\left[a_0^{b-1} - a_{cr}^{b-1}\right] \leq N\right) \qquad (18)$$

Because Eq. (18) involves two random variables, N_0 and X, a Monte Carlo simulation is employed to calculate the total life distribution, that is, the POF. The Monte Carlo simulation is conducted in Microsoft Excel, and the procedures are as follows.

Assuming u_i and v_i $(i = 1, 2, \ldots, I)$ are two random numbers between zero and unity, we can calculate the corresponding values of the two random variables N_0 and X as

$$N_0(u_i) = \tilde{N}_0 \exp\left[\Phi^{-1}(u_i)\sigma_{Z0}\right], \qquad i = 1, 2, \ldots, I \quad (19)$$

$$X(v_i) = \exp\left[\Phi^{-1}(v_i)\sigma_Z\right], \qquad i = 1, 2, \ldots, I \quad (20)$$

where Φ^{-1} is the inverse of the standard normal distribution function and I is the number of shootings in the Monte Carlo simulation. Substitution of N_0 and X in Eq. (17) leads to

$$N_t^i = N_0(u_i) + [1/(b-1) X(v_i) Q]\left[a_0^{b-1} - a_{cr}^{b-1}\right]$$

$$i = 1, 2, \ldots, I \quad (21)$$

Now define a variable as an indicator:

$$s_i = \begin{cases} 1, & \text{if} \quad N_t^i \leq N \\ 0, & \text{if} \quad N_t^i \leq N \end{cases} \qquad i = 1, 2, \ldots, I \quad (22)$$

Then the POF can be calculated as

$$\mathrm{POF}_{N_t(a_{cr})}(N) = \sum_{i=1}^{I} \frac{s_i}{I} \qquad (23)$$

The procedures just outlined are used to predict the POF, that is, the total life distribution, of the MSD specimens. Predictions using 5000 values for the random variables involved are shown in Fig. 7. Also shown in Fig. 7 is the total fatigue life distribution of the specimens at the observed first linkup. The goodness-of-fit test with an A–D statistic of $A_n^2 = 0.28$ and the level of significance above 0.25 indicate that the predicted distribution compares well with the test data, and thus the efficiency of the procedures proposed for risk analysis is verified.

Conclusions

The failure characteristics of fuselage splices containing MSD have been investigated using probabilistic analysis methods. The fatigue test data of nine noncorroded MSD specimens have provided the basis for the theoretical development of the stochastic damage growth model. The first linkup of the two cracks in a ligament was taken as the failure criterion. The total fatigue life of a lap splice with MSD was considered to be the sum of damage starting life and damage growth life. Whereas a lognormal distribution was assumed for the damage starting life, a stochastic damage growth model was developed for the growth life distribution. The analytical solution derived was used to investigate the statistical characteristics of the damage growth behavior. For the total life behavior, a Monte Carlo simulation was employed and the POF of MSD specimens was predicted. The comparisons between the analysis results and the test data verified the efficiency of the stochastic damage growth model developed and the risk assessment procedures proposed.

Based on the work conducted, the following conclusions can be drawn:

1) The test data obtained using nine MSD specimens exhibited large statistical dispersion associated with visible crack starting and growth damage accumulation, a typical phenomenon for MSD specimens. This phenomenon indicated the necessity of using probabilistic methods to investigate the failure characteristics of fuselage splices with MSD.

2) The difference between the fatigue life corresponding to the two failure modes for the specimens, that is, first linkup and final break, was found to be less than 4%. Therefore, the failure criterion based on the first linkup appears appropriate for noncorroded MSD specimens that provided a conservative estimate of the failure probability.

3) It is a useful approach to divide the total fatigue life of a structure into damage starting life and growth life when conducting risk analysis. The assumption of lognormal distribution for the damage starting life has been shown to be appropriate through comparison with the observed damage starting data. It has also been shown that the damage growth life of MSD specimens has relatively small statistical dispersion as compared to that of the damage starting life.

4) The stochastic damage growth model developed based on test data is an efficient approach for risk assessment of lap splices with MSD. The salient feature of this model is the avoidance of sophisticated fracture mechanics analysis and the simplicity as compared to risk analysis using deterministic crack growth prediction and distributed initial flaw sizes. In addition, provided sufficient test data are available, this modeling approach can be used to deal, in a relatively simple manner, with the problems with fatigue and corrosion interactions in splices.

5) The procedures outlined, using a Monte Carlo simulation in Microsoft Excel, have provided reasonably accurate predictions of the total life distribution, or POF, for the MSD specimens. Thus, the probabilistic analysis methodology proposed can be used for risk assessment of splice joints with MSD in follow-on research.

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